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ABSTRACT

The nature and evolution of one preservice secondary teacher's beliefs about mathematics are described in this paper. It provides information on how the teacher's conceptions about mathematics, teaching, and learning evolved during a secondary methods course and student teaching. The teacher communicated narrow views about the nature of mathematics and the core beliefs changed very little during the study. The main evolution during the methods course was an increased awareness of reform themes. Although the teacher refined some views during student teaching, the most significant change involved the ability to elaborate beliefs with specific classroom examples. The paper also explores the relationships between espoused beliefs and teaching decisions in the context of a reformed classroom environment. Contains 12 references. (Author/AIM)

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Developing Problem-Solving Conceptions of Mathematics: A Preservice Teacher's Experiences

by
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DEVELOPING PROBLEM-SOLVING CONCEPTIONS OF MATHEMATICS: A PRESERVICE TEACHER'S EXPERIENCES

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The nature and evolution of one preservice secondary teacher's beliefs about mathematics are described. The participant was interviewed and observed throughout the final year of her preservice program (methods course and student teaching). She communicated narrow (i.e., Instrumentalist (Ernest, 1991)) views about the nature of mathematics and her core beliefs changed very little during the study. The main evolution during the methods course was an increased awareness of reform themes. During student teaching, she refined some of her views but the most significant change involved her ability to elaborate her beliefs with specific classroom examples. Additionally, the paper explores relationships between her espoused beliefs and teaching decisions in the context of a reformed classroom environment.

This paper describes how a preservice secondary teacher's conceptions about mathematics, teaching, and learning evolved during a secondary methods course and student teaching. One characteristic of recent reform efforts in mathematics education has been an attempt to move teachers away from the sole reliance on formal lectures toward practices that support active and vocal participation by students in the learning process (NCTM, 1989). Students need opportunities to explore and communicate, and make sense of mathematical concepts and problems. Teachers, like their students, are thoughtful and reflective individuals who construct ideas about mathematics, their students' learning, and what it means to teach effectively (Cooney, 1994). Recognition of this notion has led to research focused not just on what teachers do, but also on what they think.

A different philosophy of mathematics is also reflected in the goals of reform. Mathematics is characterized by the creative and changing nature of its processes and ideas, and connections among them, not just by accurate results and infallible procedures. Teachers' views of mathematics are related to their beliefs about the teaching and learning of mathematics, and therefore play a significant role in shaping patterns of instructional behavior (Ernest, 1989; Lerman, 1990; Thompson, 1992). As such, attempting to better understand preservice teachers' beliefs about the nature of mathematics and examining their ability to change these beliefs have become important areas of study for mathematics teacher education. This paper is a description of one preservice teacher's beliefs about the nature of mathematics. Additionally, we provide insight into how her conceptions about mathematics are related to her enacted beliefs. We do so over an extended period of time (an entire academic year), enabling us to identify how her conceptions evolved over this period. These relationships are particularly relevant because they took place within the context of a preservice program aligned with the mathematics education reform movement described above.

Ernest (1989, 1991) defines three categories (Problem-Solving, Platonist and Instrumentalist) to describe beliefs individuals have about the nature of mathematics, and elaborates how these views provide a basis for teachers' conceptions of mathematics teaching and learning. According to the *Problem-Solving* view, mathematics is a continually growing field of human creation that develops through conjectures, the generation of patterns, proofs

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and questioning. It is not a finished product, but rather its results remain open to revision. Mathematics is seen as a "dynamically organized structure located in a social and cultural context" (1989, p. 250). Teachers with Problem-Solving views of mathematics are likely to think of themselves as facilitators who encourage students to pose and solve problems. Such teachers are inclined to value students' construction of their own understanding. The Platonist view suggests that mathematics is "a static but unified body of certain knowledge. . . . It is discovered, not created." This view involves a global understanding of mathematics as a consistent, connected and objective structure." A teacher with Platonist views of mathematics is likely to see his or her primary role as explainer. In general, a Platonist teacher emphasizes conceptual understanding of unified knowledge, and sees the students as "receptors of knowledge (1989, p. 250-251)." The Instrumentalist view maintains that mathematics is "an accumulation of facts, rules and skills to be used in the pursuance of some external end." Instrumentalism involves knowledge of "mathematical facts, rules and methods as separate entities" (p.250). The Instrumentalist view of mathematics is likely to be associated with an "instructor model of teaching," where teachers emphasize mastery of skills, rules and procedures and usually strictly follow a text or scheme. The student's role is to master what the teacher is telling him or her.

Often there is a disparity between espoused beliefs and what a teacher actually does in the classroom (Ernest, 1989; Cooney, 1985). In addition to views about the nature of mathematics, social context and the teacher's level of consciousness of her own beliefs can influence instructional practice (Cooney, 1994). Therefore, we believe that classroom context opportunity for reflection play significant roles in preservice teachers' development of Problem-Solving conceptions. This view not only influenced the design and implementation of the teacher education program in which Ashika participated, it also influenced the way we planned and carried out the study reported in this paper.

DESIGN

Participant and Site

Ashika completed her undergraduate degree in mathematics at a private African American college, and during the period of this study was working on a masters degree in education with secondary certification in mathematics (at a large midwestern university). She considers herself to have a strong mathematics background having been a successful student of traditional mathematics.

During the fall of 1995, Ashika was a student in a mathematics teaching methods course required for secondary mathematics certification. She participated in mathematics methods instruction that required her to explicitly connect her views, experiences, and understandings of reform themes. For example, with another student, she completed a multimedia project communicating her understanding of alternative assessment. This multimedia project also required her to reflect upon and critically analyze the practice teaching

experiences that were occurring at the same time as the methods course. The second author was course instructor.

During winter 1996, Ashika student taught in a reform-oriented classroom at a public high school. Her cooperating teacher, Ms. Stevenson, had integrated cooperative learning, technology and large group projects into her curriculum for years. Each week Ashika also attended an informal seminar with other student teachers. During these seminars, discussions centered on reform themes and student teachers' reflections about their classroom experiences. The first author supervised Ashika's student teaching experiences and led the seminar.

Data Collection

We used an ethnographic case study design (Stake, 1995) to collect information about Ashika. Data were collected between September 1995 and June 1996. We engaged Ashika in formal and informal interviews, observed her student teaching, and collected written artifacts, including surveys and assignments from the methods course and student teaching seminar. Four semi-structured interviews (approximately one hour in length) allowed Ashika to elaborate survey responses and communicate her views about mathematics, the methods course, and her practice teaching experiences. During these elaboration interviews much of our discussion arose from the participants' comments. All formal interviews were audio taped and transcribed. Selected written assignments completed in the methods course and student teaching seminar were also analyzed. During student teaching, seven classroom observations occurred. Informal interviews (length varied from 20 minutes to one hour) followed these seven observations. Finally, Ashika was observed during 12 student teaching seminars.

RESULTS AND CONCLUSIONS

This section shows how Ashika's largely Instrumentalist views about mathematics did not appear to change much, despite experience with Problem-Solving themes during her preservice program. Additionally, it addresses the issue of how her beliefs were related to her observed mathematics teaching.

Like many other preservice mathematics teachers (e.g., Owens, 1987; Wilson, 1994), at the beginning of the methods course most of Ashika's communications were consistent with an Instrumentalist view of mathematics. She saw mathematics as being sequential, building from a foundation of arithmetic and algebra. In order to understand the more advanced areas of mathematics, Ashika claimed that it was essential to know the basics and how to apply them. As she explained during our first interview:

What you learned in elementary school is basic. I mean, you get this complex calculus problem and it's all going to boil down to you adding, subtracting. So if you get stuck at the foundation, then you can't move on....I don't see math as being very complicated...the concepts might be complex, but once you muddle through it and get to it, all I'm doing is adding. But, most people don't see that.

Similarly, on several occasions Ashika referred to having to practice and memorize procedures in order to learn new material. According to Ashika, formulas are "a step-by-step kind of thing to come to an answer . . . if you have this formula, then you're sure, you know what

you're doing, you're sure to come up with the right answer." To Ashika, doing mathematics mainly involved following a set of rules or formulas that will always lead you to the right answer.

However, Ashika did not see mathematics as consisting exclusively of these formulas. During her experiences in high school geometry and as a student of college mathematics, she discovered what she labels the "theory part," or proofs, of mathematics. She described why these experiences were unsettling to her:

Because, when you're younger, you think "Oh math is adding" . . . and you think that it's some application or some process or whatever, and then . . . It happened to me twice. It happened to me in geometry and then it happened to me when I got into my first math theory class, that you think, "OK, I can do calculus or whatever," and then someone goes, "Well prove this." And then, and they're telling you, "This is the true math. That's just baby stuff, and anybody can do that. But, if you can do this, then you are really a mathematician." . . . That's why it shocked me because I could do one, but I wasn't very. I had to work hard at doing the other. . . . Most people, they don't know that side of math. . . . They think of math as being all formulas. . . . That's all you do is number crunching.

Although she acknowledged that there was a non-"number-crunching" aspect of mathematics, she saw the theory part as being utilized mainly by mathematicians, and not understood or used by most people. While as a student she had to develop and understand proofs, she did not think that it was important for her students to understand this part of mathematics.

Like other students, Ashika claimed that her conceptions had not changed very much during the methods course. While we detected some change, the nature of that change was related more to an increased recognition of and familiarity with issues that she had not fully articulated before. For example, at the end of the course Ashika reiterated her beliefs that building a strong foundation is important at the high school level, and she continued to see arithmetic as being an essential "skill that you need no matter what you do." However, she also expressed that the basics need to become automatic so that students can focus on processes, concepts, and applications. According to Ashika, students need to master the basics before they are "ready to do the conceptual stuff, because they won't have to think about the manipulation stuff." As she stated, "How can you understand how to apply something if you don't understand what it is?" While this comment illustrates that Ashika still held the core belief that mathematics is mainly an accumulation of rules and skills to be mastered (an Instrumentalist view), her reference to applications, problems and concepts suggests that, at least to a small extent, she saw mathematics as being connected (Platonist).

Her movement toward a Platonist view of mathematics is further illustrated by one of her survey responses (and subsequent elaboration). Ashika discussed why mathematics is useful (to describe natural phenomena, explain why something happened, find answers to problems, and to predict future events). She spoke less of finding answers and using formulas, and more of students "thinking about what they are doing," and analyzing the process (i.e., the steps that one uses to solve problems). She began to articulate a belief that mathematics is an activity that enables students to think critically.

Throughout her student teaching experience, Ashika's views of mathematics influenced her practice and facilitated her reconciliation of some of her beliefs about the teaching and learning of mathematics. For example, at the end of her student teaching, Ashika talked about a lesson that she taught during a logic unit in geometry. She asked her students to make *Kool-Aid* from a recipe and discuss their problem solving process. She particularly enjoyed this lesson because she felt it "broke up the monotony, . . . involved something concrete that everyone could relate to," and helped the students become engaged and make "connections." When explaining the connections she hoped students would make, she discussed how she tried to convey to the students that to be efficient when following a recipe, you need to have all the ingredients ready in advance. Ashika then elaborated on this point and related her lesson to problem solving in general:

So, you need the skills to go to a problem, where you have all the ingredients. And like, an ingredient could be like just basic background information, formulas, and things like that, that all help in the problem solving process.

Even though this comment occurred during a discussion about problem solving, the statement reveals her underlying belief that the foundation or essential aspect of mathematics is its formulas. Similarly, in the student teaching seminar she described how her students needed to be shown how to do two-column and paragraph proofs before they could successfully do proofs themselves. In this example, the foundation consisted not of procedures, but of the skills that were necessary for students to be able to focus on the idea and process of proof.

Throughout her student teaching, we observed Ashika reconciling her core conceptions of mathematics (e.g. mathematics builds from a foundation, which students must first master before they learn concepts) with her beliefs about how to teach and how students learn. For example, she struggled with the issue of how much she should tell her students, and when it was appropriate to let them explore: "What should I do if they are not getting it? How much to tell? What else can I do?" This is an example of a theme she did not mention very much during the methods course (although it was discussed at considerable length in the course). At the end of student teaching, Ashika explained what she considered her role as teacher to be.

In some form or another, to transfer knowledge, or to help students gain knowledge. It depends on exactly what you are doing. Some things students just have to be told, and then you move from there. But, there are certain things that you can explore together and that they can be part of, like the uncovering of the material and concepts, and things like that.

Here we see a recognition that concepts are not only important, but that students should come to understand those concepts through exploration. It is not clear whether Ashika's recognition of the importance and value of exploration occurred because she taught in a classroom where student exploration occurred on a regular basis, but these circumstances allowed her to connect this realization to actual teaching experience. We emphasize however, that Ashika's acknowledgment of the value of student exploration did not signify a shift in her core beliefs about the nature of mathematics. She did not even recognize that her core beliefs conflicted

with Ms. Stevenson's. Ashika attributed most of Ms. Stevenson's innovative methods and the circumstances (e.g., teaching with no textbook) not to a difference in philosophy, but to Ms. Stevenson's experience

DISCUSSION

Ashika's case illustrates how impervious preservice teachers' beliefs can be to change. Despite being inundated with reform themes during her preservice experience, Ashika maintained largely Instrumentalist views of mathematics. Ashika claimed that during her student teaching she lectured very little. Although our observations verified this fact, most of Ashika's comments suggest that she will do more lecturing when she becomes a teacher. Ashika generally followed Ms. Stevenson's mostly innovative organizational and instructional formats. This may account for the discrepancies we observed. This raises two questions: (1) Is it necessary for teachers to hold Problem-Solving views of mathematics to teach in innovative ways?, and (2) Does experience with innovation during practice teaching guarantee that preservice teachers will adopt Problem-Solving conceptions? Evidence from this study suggests that the answer to both questions is no.

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